Chapter Six

Neo-Riemannian Transformations and Wyschnegradsky's DC-scale

Over the last twenty years, there have been a number of speculative theoretical articles that consider generalized algebraic models of intervals, chords, and scales. Even though these articles do not directly address microtonal music, their models can be applied to a variety of equal-tempered systems. For example, Carey and Clampitt's well-formed scales are found in 5, 7, 12, 17, 29, 41, and 53 equal divisions of the octave.[1] And Rapoport examines the acoustic properties of thirds and fifths (and their nearest equivalents) in equal-tempered systems as large as 284 divisions of the octave.[2] In Chapter 5, I show how Clough, Engebretsen, and Kochavi's taxonomy of scale properties enriches our understanding of Wyschnegradsky's DC-scale.[3] In this chapter, I examine Richard Cohn's

[1] Norman Carey and David Clampitt, "Aspects of Well-Formed Scales," *Music Theory Spectrum* 11/2 (1989), 187-206.
[2] Paul Rapoport, "The Structural Relationships of Fifths and Thirds in Equal Temperaments," *Perspectives of New Music* 37/2 (1993), 351-90.
[3] John Clough, Nora Engebretsen, and Jonathan Kochavi, "Scales, Sets, and Interval Cycles: A Taxonomy," *Music Theory Spectrum* 21/1 (1999), 74-104.

generalized parsimonious trichord and apply his model of neo-Riemannian transformations to Wyschnegradsky's DC-scale.

In "Neo-Riemannian Operations, Parsimonius Trichords, and Their *Tonnetz* Representations," Cohn formulates algebraic definitions of the canonic neo-Riemannian transformations P, L, and R (which he labels as the PLR family) that accommodate an infinite number of microtonal schemes.[4] The generalized PLR family operates on a generalized trichord derived from the conventional consonant triad. Cohn looks at the properties of his generalized trichord in c=24; two of his examples deal specifically with quarter-tone trichords. Although the conventional consonant triad has profound musical significance, the generalized quarter-tone trichord that Cohn derives from it does not. I draw this conclusion because it does not appear as a structurally significant harmonic entity in any of the quarter-tone music I have studied. Furthermore, as I demonstrate in the music of Blackwood, Hába, Ives, and Wyschnegradsky, significant chords in quarter-tone music are more likely to be tetrachords than trichords.

In this chapter, I examine Cohn's generalized PLR family and the generalized trichord that supports these transformations in the context of the

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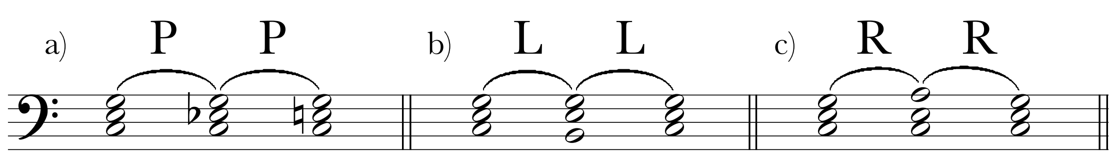
[4] Richard Cohn, "Neo-Riemannian Operations, Parsimonius Trichords, and Their *Tonnetz* Representations," *Journal of Music Theory* 41/1 (1997), 1-66 passim.

quarter-tone universe. Because Cohn's generalized transformations apply only to trichords, I begin my examination of quarter-tone transformations by first considering quarter-tone trichords. Next, I apply the transformations in the PLR family to the tonic tetrachord of Wyschnegradsky's DC-scale. I then look at Cohn's PLR operator cycles, comparing the cycles that operate on the generalized trichord with those that operate on Wyschnegradsky's tetrachord. In Chapter 5, I show that the DC-scale and its tonic chord (in both large and small configurations) can support prolongations of tonic harmony. In this chapter, I show that Wyschnegradsky's chord supports its own PLR family.

Conventional Neo-Riemannian Transformations

Before examining Cohn's generalized PLR family, I will briefly review the conventional neo-Riemannian transformations P, L, and R. Although these transformations are well known among theorists, a review of their characteristics will help in applying the PLR family to the DC-scale. Example 6.1 presents the conventional PLR family. The *Parallel* transformation, or P, transforms a triad into its modal opposite. Example

6.1a shows that P transforms C–major into C–minor and transforms C–minor to C–major. The *Leittonwechsel* transformation, or L, transforms a major triad by exchanging its root for the pitch one semitone below it, and it transforms a minor triad by exchanging its fifth for the pitch one semitone above it. Example 6.1b shows that L transforms C–major into E–minor and transforms E–minor to C–major. The *Relative* transformation, or R, transforms a major triad by exchanging its fifth for the pitch one whole tone above it, and transforms a minor triad by exchanging its root for the pitch one whole tone below it. Example 6.1c shows that R transforms C–major into A–minor and transforms A–minor to C–major. A consequence of R is that the two triads involved, the "before" and "after" triads, are the tonic triads of a pair of relative major and minor scales.



Example 6.1: Conventional PLR family

The three transformations that make up the PLR family share some common characteristics. All three transformations effect a change of mode; that is, they transform major triads into minor triads, and minor triads into major triads. All three transformations are involutions: when applied twice in succession, they produce the original triad. (That is, the three transformations in Example 6.1 all transform a C–major triad into a different minor triad and then back to C–major again.) All three transformations are characterized by smooth voice-leading: two common tones are retained, while the third voice proceeds by step (one semitone for P and L, one whole tone for R). Cohn refers to this smooth voice-leading as "parsimonious voice-leading."[5]

Cohn's definition of generalized parsimonious voice-leading between triads requires two conditions: (1) two voices are retained as common tones, and (2) the third voice proceeds by an interval no greater than twice the smallest available interval in a given tuning.[6] The transformations in the conventional PLR family satisfy both conditions; in Example 6.1, each of the three

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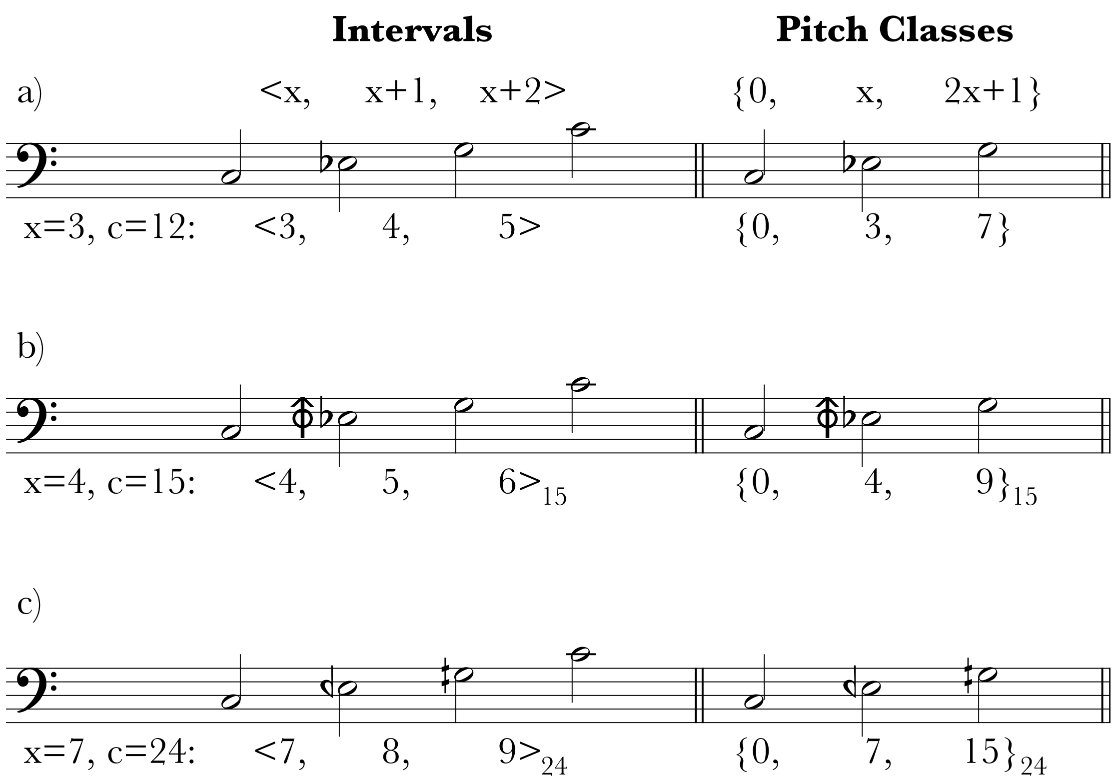
[5] Cohn, "Neo-Riemannian Operations, Parsimonius Trichords, and Their *Tonnetz* Representations," 1.

[6] We normally characterize the voice-leading in the PLR family as stepwise. In c=12, twice the smallest interval (one semitone), is equivalent to one whole tone, the largest interval that would ordinarily be considered a scale-step. For cardinalities other than c=12, however, there is no defined scale, and so Cohn's generalized definition of parsimonious voice-leading must rely on interval size.

transformations preserve two common tones, while the moving voice proceeds by int 1 (the smallest possible interval in c=12) in the case of P and L, and int 2 (two semitones or twice the smallest possible interval) in the case of R.

Cohn's Generalized Parsimonious Trichord

Cohn's "parsimonious trichord" is a trichord capable of parsimonious voice-leading under the PLR family of transformations. The only trichord meeting this definition in c=12 is the conventional consonant triad. Cohn observes that this triad has successive intervals of <3, 4, 5> (measured in semitones). Example 6.2a shows that this interval structure describes the conventional minor triad. From this structure, Cohn derives the following generalization: for any cardinality c=3x+3, there is a parsimonious trichord composed of successive intervals <x, x+1, x+2>. For c=15, the parsimonious trichord has successive intervals of <4, 5, 6>$_{15}$, making it equivalent to Easley Blackwood's minor triad in 15-note equal temperament (see Chapter 2). In c=24, Cohn identifies a parsimonious trichord made up of successive intervals of $<7, 8, 9>_{24}$ (he does not use a decimal notation to identify pitch-classes).[7]



Example 6.2: Instances of Cohn's generalized parsimonious trichord

From this generalized trichord and his observations about conventional neo-Riemannian transformations, Cohn defines the generalized PLR family of transformations summarized in Table 6.1. Example 6.3 provides musical

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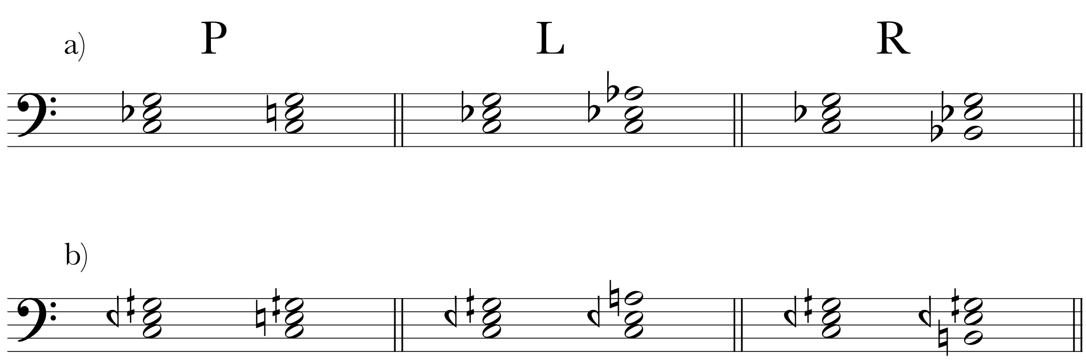
[7] My notation identifies the interval succession $<7, 8, 9>_{24}$ as $<3.5\ 4.0\ 4.5>$. However, Cohn's algebraic definitions are more convenient if we use mod 24 integers in place of my decimal notation.

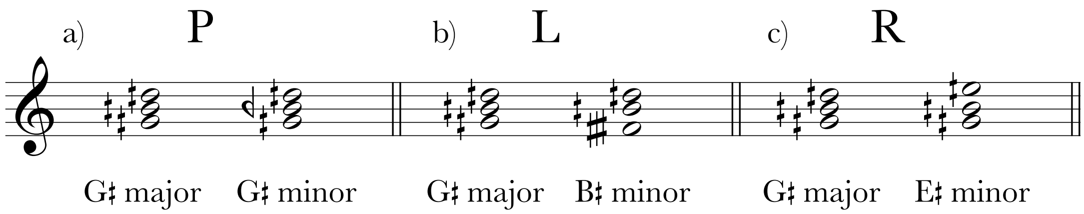
realizations of Cohn's algebraic definitions. For example, Cohn's generalized L transforms a trichord of the form {0, x, 2x+1} into a trichord of the form {0, x, 2x+2}. For c=12 (x=3), therefore, L transforms {0, 3, 7} into {0, 3, 8}. As Example 6.3a shows, L transforms a C–minor triad into an A♭–major triad, which is precisely the conventional *Leittonwechsel* transformation. For c=24 (x=7), L transforms $\{0, 7, 15\}_{24}$ into $\{0, 7, 16\}_{24}$, represented in Example 6.3b as a transformation from {C♮, E𝄳, G‡} to {A♮, C♮, E𝄳}.

		x=3, c=12		**x=7, c=24**	
		from	**to**	**from**	**to**
P	{0, x, 2x+1}⇒{0, x+1, 2x+1}	{0, 3, 7}	{0, 4, 7}	$\{0, 7, 15\}_{24}$	$\{0, 8, 15\}_{24}$
L	{0, x, 2x+1}⇒{0, x, 2x+2}	{0, 3, 7}	{0, 3, 8}	$\{0, 7, 15\}_{24}$	$\{0, 7, 16\}_{24}$
R	{0, x, 2x+1}⇒{3x+1, x, 2x+1}	{0, 3, 7}	{10, 3, 7}	$\{0, 7, 15\}_{24}$	$\{22, 7, 15\}_{24}$

Table 6.1: Cohn's generalized PLR family



Example 6.3: Cohn's generalized PLR family: a) c=12; b) c=24



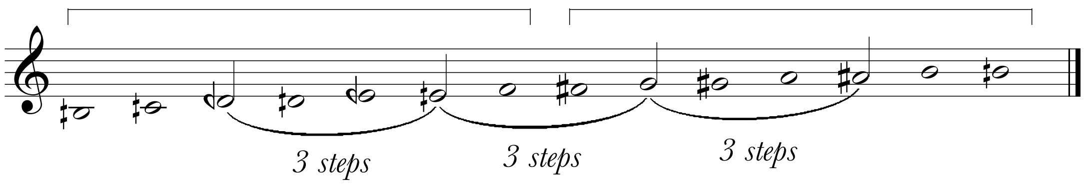
Example 6.4: Conventional PLR family with quarter-tone pitches

Cohn's definition leads to the quarter-tone PLR family shown in Example 6.3b above, but this is not the only valid realization of P, L, and R in quarter-tone space. In 24-note equal temperament (and, indeed, any equal temperament where c≡0, mod 12), we can find conventional equal-tempered major and minor triads. In Chapter 1, I demonstrate that c=24 generates 24 major and 24 minor triads; half of these have conventional roots, while the other half have quarter-tone roots, and it is still possible in c=24 to understand the conventional PLR family operating on these conventional triads in familiar ways. For example, the conventional L operator transforms C–major into E–minor. Since we can situate both the C–major and the E–minor triad in quarter-tone space, this transformation, as notated, is indistinguishable from its conventional counterpart in c=12. Furthermore, we can apply the conventional PLR family to conventional triads with quarter-tone roots. Example 6.4 shows the conventional PLR family applied to a G‡–major triad. Following the voice-leading patterns of the conventional transformations in Example 6.1 above, P transforms G‡–major to G‡–minor, L transforms G‡–major to B‡–minor, and R transforms G‡–major to E‡–minor. In addition to Cohn's generalized PLR-family, then, c=24 supports the conventional PLR-family, which leads me to conclude that depending upon the chord-type we choose to transform, there can be multiple, co-existing PLR families in quarter-tone space. As seen in Chapter 5, the primary chord-type in Wyschnegradsky's *24 Preludes* is neither the conventional consonant triad nor Cohn's generalized parsimonious trichord, but rather the tonic tetrachord. I now define a new PLR family that operates on Wyschnegradsky's small tonic tetrachord.[8]

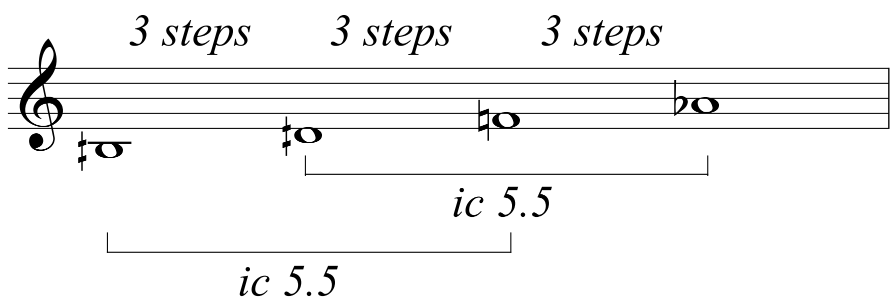
Neo-Riemannian Transformations in the DC-Scale

To simplify the following discussion, I define a generic DC-chord as a tetrachord derived from the DC-scale. The root of the generic DC-chord must be a member of the DC-scale, and the upper chord-tones are the first three pitches of a cycle of three scale-steps leading up from the root.

[8] Because every large tonic chord has identical pitch content to the small tonic chord of the DC-scale whose tonic is adjacent on the circle of fourths (see Chapter 5), I consider only small tonic chords in this study.



Example 6.5: Generic DC-chord structure

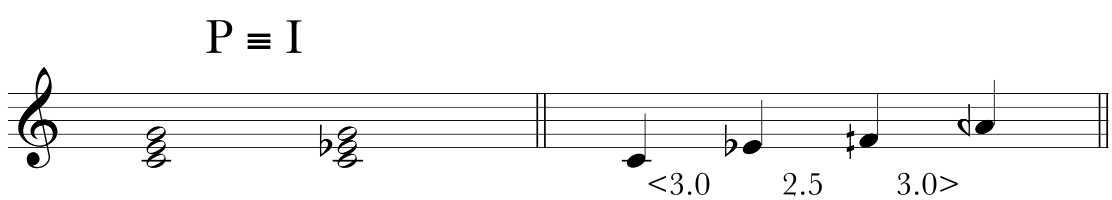


Example 6.6: Root position B𝄲 DC-major chord

Example 6.5 shows an arbitrary generic DC-chord {D𝄳, E𝄲, G, A♯} derived from the DC-scale on B𝄲, with a root of D𝄳. The specific quality of the DC-chord will depend upon the specific sizes of its component intervals, which in turn will depend upon the DC-chord's location within the DC-scale. I call the specific DC-chord whose root is the tonic of the DC-scale a "DC-major" chord. Example 6.6 shows the tonic chord from Wyschnegradsky's Prelude No. 14 (see Chapter 5), which we can now characterize as a "B𝄲 DC-major chord." I define the root position of the DC-chord as the configuration based on the cycle of three DC-scale steps (shown in Example 6.5) and the root of the DC-chord as the lowest pitch in this configuration (in this case B𝄲). Owing to the interval structure of the DC-scale, the successive intervals of the arpeggiation of a root position DC-chord will always be int 2.5 or int 3.0. The interval structure of the major DC-chord is <3.0 2.5 3.0>.

In order to define a PLR family that operates on DC-chords, I first assume that the DC-scale is analogous to the major scale (a reasonable assumption, given the DC-scale's characteristics outlined in Chapter 5); for convenience, I call Wyschnegradsky's DC-scale a "major DC-scale." I further assert that any DC-chord (major or otherwise) has a root and, consequently, a configuration that corresponds to root position. A DC-chord retains its root identity regardless of its specific inversion, just as a C–major chord retains its root of C regardless of its specific inversion. Because the PLR family generates inverted DC-chords, we must be able to identify the root of any arbitrary configuration of a particular DC-chord. The root of the DC-chord will always be the lowest pitch when the members of the DC-chord are arranged in the cycle of three scale-steps (the form shown in Example 6.6 above).

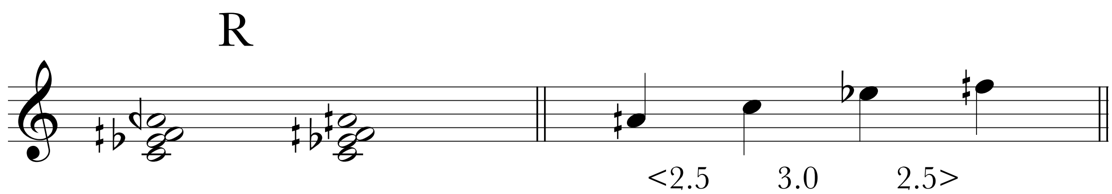
The PLR family that operates on the DC-chord must share significant musical characteristics with the conventional PLR family, such as common-tone retention and parsimonious voice-leading. In addition, just as the conventional P, L, and R always transform a major triad into a minor triad, these three transformations should transform the major DC-chord into some other chord type that we can arbitrarily label a “minor” DC-chord. They should also be involutions; when applied twice in succession, they should produce the original DC-chord regardless of whether it is major or minor. Because Wyschnegradsky did not describe distinct major and minor DC-chords, I will determine the structure of a hypothetical minor DC-chord by defining the PLR family that operates on the major DC-chord.



Example 6.7: Equivalence of conventional P and I; inversionally symmetrical DC-chord

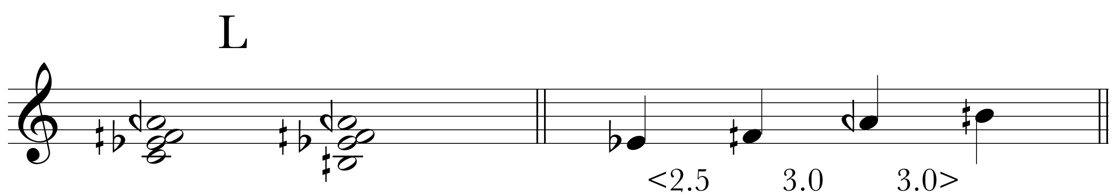
To define a P transformation that operates on the major DC-chord, we must decide what characteristics of P can be preserved when the DC-chord replaces Cohn’s generalized trichord. Unfortunately, it is difficult to find an aspect of P that can be applied to the DC-chord. Cohn’s generalized P cannot be applied directly because his algebraic definition will not accommodate tetrachords. We can observe that the generalized P is equivalent to a pitch-class inversion that preserves the highest and lowest pitches of the chord, but this characteristic cannot be applied to the major DC-chord. As shown in Example 6.7, the transformation that turns C–major into C–minor is equivalent to a pitch-class inversion of {C, E, G}, but the major DC-chord, with an interval structure of <3.0 2.5 3.0>, is inversionally symmetrical. Pitch-class inversion trivially transforms any major DC-chord into an identical major DC-chord and not some hypothetical minor DC-chord. The conventional behaviour of P, which transforms a triad into its modal opposite, does not appear to be applicable since there seem to be no criteria for establishing modally distinct major and minor DC-chords.

However, even though there is little obvious justification for defining separate major and minor versions of the DC-chord, by assuming that Wyschnegradsky's DC-scale is analogous to the major scale, one can deduce the structure of a hypothetical minor DC-scale. I start by determining plausible forms of L and R as applied to the DC-chord. Once I have established an equivalent for R, I propose a pair of relative DC-scales, one major and one minor. I then determine P by comparing the interval structures of these two proposed scales.



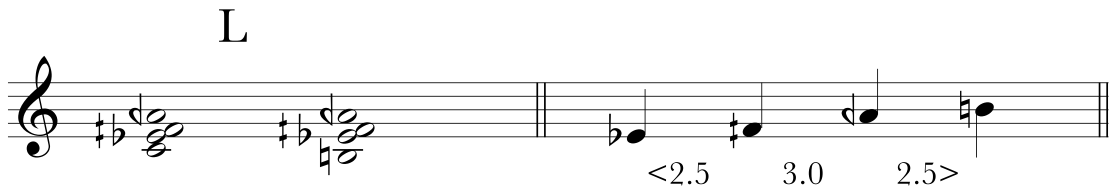
Example 6.8: Diatonic R in the DC-scale

Example 6.8 shows an R-like transformation applied to the DC-chord that shares important musical characteristics with the conventional R transformation. Like the conventional R transformation, the transformation in Example 6.8 changes one note (here, A𝄳 becomes A𝄲), and preserves all the remaining members of the chord as common tones. The conventional R operates on a major triad by preserving the lowest two pitches (root and third) and replacing the highest pitch (the fifth) with the pitch that lies one diatonic scale-step above it. The R-like transformation in Example 6.8 acts on the root-position C♮ DC-major chord {C♮, E♭, F𝄲, A𝄳} by preserving the lowest three pitches while replacing the highest pitch, A𝄳, with A𝄲, the pitch that lies one diatonic scale-step above A𝄳 in the DC-scale on C♮. I call this transformation "diatonic R" not only because the A𝄳 is replaced with its diatonic upper neighbour, but also to distinguish this transformation from a "chromatic R" which I define below. Diatonic R, therefore, transforms {C♮, E♭, F𝄲, A𝄳} into {C♮, E♭, F𝄲, A𝄲}. Rearranging the resultant chord in root position (as in the right half of Example 6.8) gives a chord with a root of A𝄲 and an interval structure of <2.5 3.0 2.5>. In other words, diatonic R transforms the C♮ DC-major chord into an A𝄲 DC-chord of an unknown type.

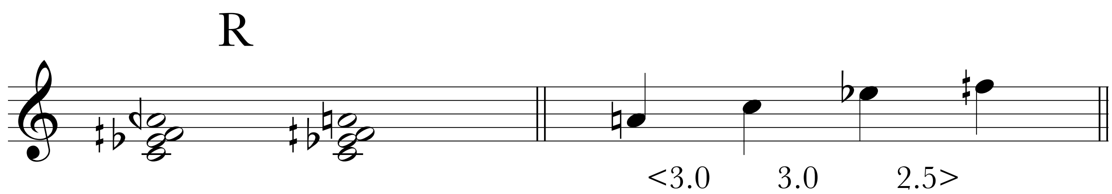


Example 6.9: Diatonic L in the DC-scale

As with the diatonic R transformation above, I define a "diatonic L" transformation by observing the musical characteristics of the conventional L transformation and applying a similar procedure to the major DC-chord. The conventional L operates on a major triad by preserving the highest two pitches (third and fifth) and replacing the lowest pitch (the root) with its diatonic lower neighbour. The diatonic L transformation (shown in Example 6.9), acts on C♮ DC-major by preserving the highest three pitches while replacing the lowest pitch, C♮, with its diatonic lower neighbour, B𝄲. The resultant chord has a root of E♭ and an interval structure of <2.5 3.0 3.0>.

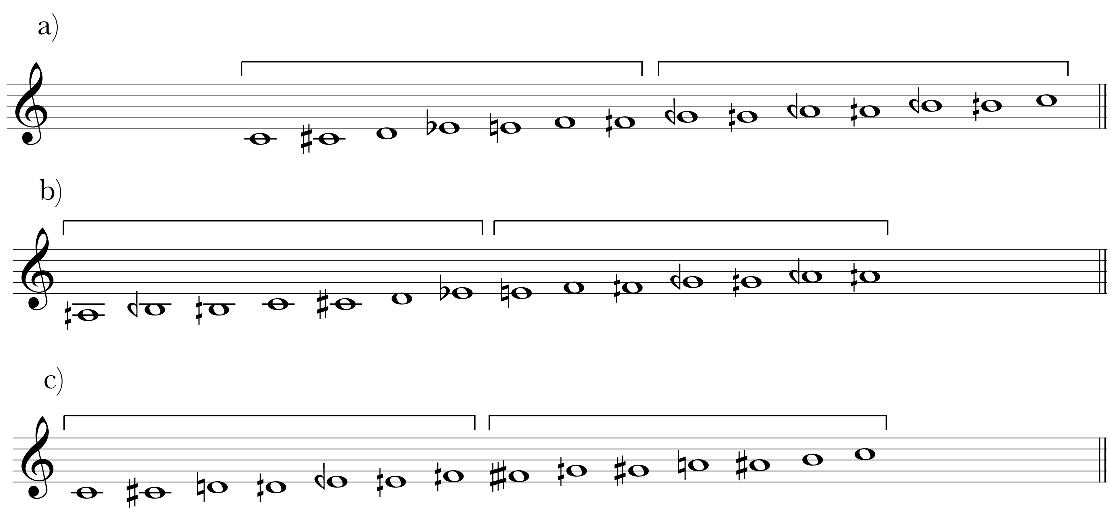


Example 6.10: Chromatic L in the DC-scale



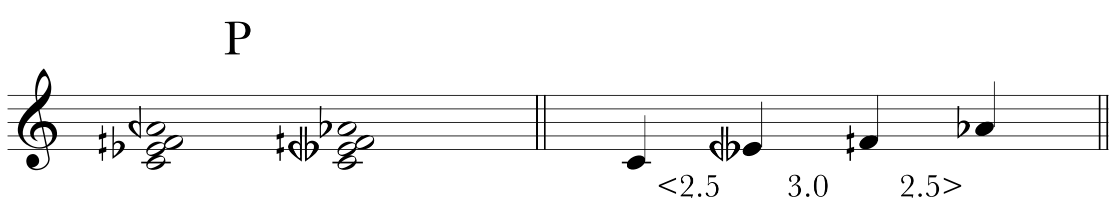
Example 6.11: Chromatic R in the DC-scale

Comparing Example 6.8 to Example 6.9 suggests that diatonic R and diatonic L cannot be members of the same PLR family because the resultant chords of the two transformations are not transpositionally equivalent. Therefore, the two transformations do not produce a consistent minor-type DC-chord. However, if I modify diatonic L as in Example 6.10, so that C♮ is replaced by its chromatic lower neighbour, B♮ (in place of the diatonic B‡), a chord results with an interval structure of <2.5 3.0 2.5>, which is transpositionally equivalent to the chord resulting from diatonic R in Example 6.8 above. Because this chromatic L transformation and diatonic R produce transpositionally equivalent results when applied to the DC-major chord, I conclude that these two transformations are members of the same PLR family. I designate the resultant DC-chord, with its <2.5 3.0 2.5> structure, as the DC-minor chord. Having now determined R, L, and the structure of the DC-minor chord, I now can define the P transformation. (It also is possible to define a "chromatic R" transformation in which the diatonic upper-neighbour A𝄲 is replaced with A♮ [see Example 6.11]. The resultant chord, with an interval structure of <3.0 3.0 2.5>, does not match any of the DC-chords discussed above.)



Example 6.12: DC-scales: a) C♮ DC-major; b) A𝄲 DC-minor; c) C♮ DC-minor

The two triads involved in any conventional R transformation represent the tonic triads of a relative pair of major and minor scales. If diatonic R transforms C♮ DC-major into A𝄲 DC-minor, then we can posit that these two DC-chords are tonics of a pair of relative scales. The structure of C♮ DC-major is known (Example 6.12a), but what is the structure of A𝄲 DC-minor? Just as the conventional A–minor scale can be thought of as a rotation of its relative major, the C–major scale, the A𝄲 DC-minor scale can be thought of as a rotation of the C♮ DC-major scale. The scale in Example 6.12b starts on A𝄲 and ascends through one full octave, using as its scale steps the pitches of the C♮ DC-major scale. I call this new scale the A𝄲 DC-minor scale. If I transpose the scale in Example 6.12b so that its tonic becomes C♮, the result is the C♮ DC-minor scale shown in Example 6.12c.



Example 6.13: The P transformation in the DC-scale

The two triads related by the conventional P transformation always represent the tonic triads of a pair of parallel major and minor scales. If I compare the tonic tetrachords of the C♮ DC-major and C♮ DC-minor scales, I can define P as the transformation that transforms C♮ DC-major {C, E♭, F𝄲, A𝄳} into C♮ DC-minor {C, E𝄳♭, F𝄲, A𝄳}. As shown in Example 6.13, this P transformation is different from the others because it features two moving voices (A𝄳–A♭ and E♭–E𝄳♭); only two pitches, C♮ and F𝄲, are retained as common tones.[9]

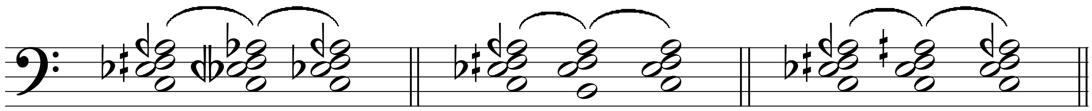
Root	Chord	Intervals	Type
C♮	{C♮, E♭, F𝄲, A𝄳}	<3.0 2.5 3.0>	DC-major
C♯	{C♯, E♮, G𝄳, A𝄲}	<3.0 2.5 3.0>	DC-major
D♮	{D♮, F♮, G𝄲, B𝄳}	<3.0 2.5 3.0>	DC-major
E♭	{E♭, F𝄲, A𝄳, B𝄲}	<2.5 3.0 3.0>	diatonic L
E♮	{E♮, G𝄳, A𝄲, C♮}	<2.5 3.0 2.5>	DC-minor
F♮	{F♮, G𝄲, B𝄳, C♯}	<2.5 3.0 2.5>	DC-minor
F𝄲	{F𝄲, A𝄳, B𝄲, D♮}	<3.0 3.0 2.5>	chromatic R
G𝄳	{G𝄳, A𝄲, C♮, E♭}	<3.0 2.5 3.0>	DC-major
G𝄲	{G𝄲, B𝄳, C♯, E♮}	<3.0 2.5 3.0>	DC-major
A𝄳	{A𝄳, B𝄲, D♮, F♮}	<3.0 2.5 3.0>	DC-major
A𝄲	{A𝄲, C♮, E♭, F𝄲}	<2.5 3.0 2.5>	DC-minor
B𝄳	{B𝄳, C♯, E♮, G𝄳}	<2.5 3.0 2.5>	DC-minor
B𝄲	{B𝄲, D♮, F♮, G𝄲}	<2.5 3.0 2.5>	DC-minor

Table 6.2: Chord types in the C♮ DC-major scale

[9] As stated above, Cohn's definition of parsimonious voice-leading in trichords involves two conditions: two voices retained as common tones, and one moving voice. Clearly it is impossible for a tetrachord to satisfy both conditions. My compromise is to allow P to satisty the first condition, and R and L to satisfy the second condition.

Table 6.2 shows a relationship between the DC-chords and the diatonic DC-scale that recalls a similar relationship in the conventional major scale. In the major scale, there are seven triads: three major, three minor, and one diminished; the two most frequently occurring triad types are the major and minor triads. In the diatonic DC-scale, the two most frequently occurring chord types are the DC-major chord and the DC-minor chord. Table 6.2 enumerates the DC-chord types that are built upon each of the thirteen distinct scale-steps in the C♮ DC-major scale. There are six DC-major chords (with roots of C♮, C♯, D♮, G𝄳, G𝄲, and A𝄳), five DC-minor chords (with roots of E♮, F♮, A𝄲, B𝄳, and B𝄲), and two DC-chords that are neither major nor minor. The E♭ DC-chord is transpositionally equivalent to the chord type generated when diatonic L is applied to the DC-major chord; the F𝄲 DC-chord is transpositionally equivalent to the chord type generated when chromatic R is applied to the DC-major chord.

	a) DCP	b) DCL	c) DCR



root:	C C C	C E♭ C	C A♮ C
moving voices:	two	one	one
interval:	int 0.5 (both voices)	int 1.0	int 1.0

Example 6.14: The DC PLR family

Example 6.14 summarizes the three transformations of the PLR family applied to the DC-major chord. I call these transformations DCP, DCL, and DCR to distinguish them from Cohn's generalized P, L, and R. The three DC-transformations, like their conventional counterparts, feature smooth voice-leading. DCL and DCR preserve three common tones while the fourth voice proceeds by semitone (int 1.0). DCP preserves only two common tones, while the other two tones each proceed by quarter tone (int 0.5). The total voice-leading for each transformation is int 1.0, expressed by a single moving voice for DCL and DCR, and shared between two moving voices for DCP.

Cohn's generalized definition of parsimony allows for only one moving voice that must proceed by not more than two of the smallest available intervals, and so it would appear that DCP does not exhibit parsimonious voice-leading. However, Cohn's definition applies only to trichords; tetrachordal parsimony cannot conform precisely to his definition. In a PLR-family transformation applied to a parsimonious trichord, two voices are retained as common tones; likewise, in the case of DCP, two voices also are retained as common tones. Therefore, I consider DCP to possess parsimonious voice-leading because of its common-tone retention and the smooth voice-leading of the two moving voices, which proceed by quarter-tone, the smallest available interval in c=24.

Compound Transformations and Transformation Cycles

	Root Motion			
	from DC-major to DC-minor		*from DC-minor to DC-major*	
DCP	int 0.0	C ⇒ c	int 0.0	c ⇒ C
DCL	up int 3.0	C ⇒ e♭	down int 3.0	c ⇒ A
DCR	down int 2.5	C ⇒ a𝄲	up int 2.5	c ⇒ D𝄲

Table 6.3: Root motion in the DC PLR family

In his discussion of the parsimonious trichord, Cohn describes compound transformations that result when two or more of the PLR operations are applied in succession. The compound transformations that result from the DC versions of P, L, and R show some similar behaviour when compared to their counterparts in Cohn's system. In order to simplify the following discussion, I use a streamlined notation to describe the effects of DCP, DCL, and DCR. To this point, I describe these transformations in terms of voice-leading and common-tone retention, but I also could easily describe them in terms of root motion and the change of mode, as shown in Table 6.3. DCP will always effect a change of mode from DC-major to DC-minor, but the root of the DC-chord will remain unchanged. (In Table 6.3 and the discussion that follows, I identify both conventional triads and DC-chords by their roots, with uppercase note names signifying major and lowercase note names signifying minor.) Under DCL, a DC-major chord is transformed into a DC-minor chord whose root is int 3.0 higher (for example, C DC-major becomes e♭ DC-minor), and a DC-minor chord is transformed into a DC-major chord whose root is int 3.0 lower (c DC-minor becomes A DC-major). Under DCR, a DC-major chord is transformed into a DC-minor chord whose root is int 2.5 lower, and a DC-minor chord is transformed into a DC-major chord whose root is int 2.5 higher.

P L P L P L

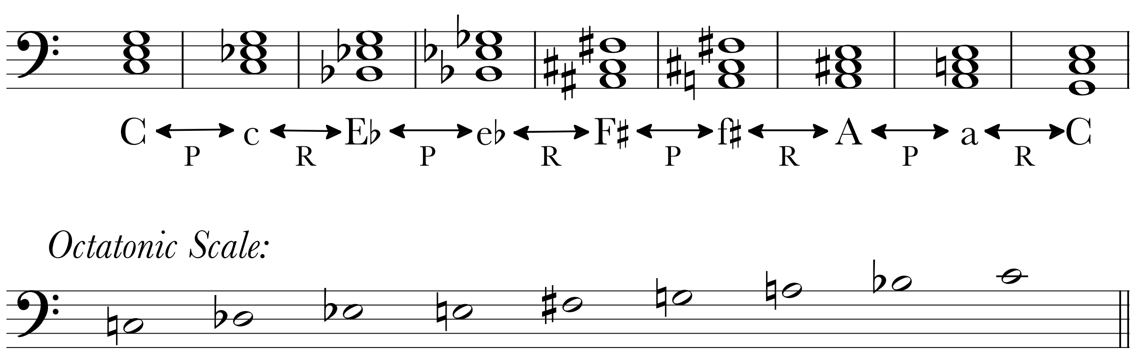
C ⇔ c ⇔ A♭ ⇔ a♭ ⇔ E ⇔ e ⇔ C

Example 6.15: Conventional PL-cycle

P R P R P R P R

C ⇔ c ⇔ E♭ ⇔ e♭ ⇔ F♯ ⇔ f♯ ⇔ A ⇔ a ⇔ C

Example 6.16: Conventional PR-cycle



Example 6.17: Conventional PR cycle and related octatonic scale

A binary transformation applies two members of the PLR family in succession. After multiple iterations, a binary transformation generates a cycle that traverses a series of distinct triads before returning to its starting point. The binary transformation PL generates the six-chord cycle shown in Example 6.15.[10] Its retrograde binary transformation LP produces an

[10] This cycle, which Cohn has named the "hexatonic cycle," is discussed in greater detail below. See Example 6.25.

equivalent cycle, but traverses the triads in reverse order. (The double arrowheads in Example 6.15 signify that the transformations may be read either left-to-right or right-to-left.) Pairing P with R generates the cycle in Example 6.16. The eight triads of this cycle, four major and four minor, are the set of eight consonant triads that together make up the pitches of the octatonic scale shown in Example 6.17. Pairing L with R generates a long cycle that visits all 24 consonant triads before returning to its starting point. As Example 6.18 shows, the LR-cycle can be separated into a series of fifth-related major triads {C, G, D, A, …} that alternates with a series of fifth-related minor triads {e, b, f♯, c♯, …}.

L R L R L R L R

C ⇔ e ⇔ G ⇔ b ⇔ D ⇔ f♯ ⇔ A ⇔ c♯ ⇔ …

Example 6.18: Conventional LR-cycle

DCP DCL DCP DCL DCP DCL DCP DCL

C ⇔ c ⇔ A ⇔ a ⇔ F♯ ⇔ f♯ ⇔ E♭ ⇔ e♭ ⇔ C

Example 6.19: DC PL-cycle

DCP DCR DCP DCR DCP DCR DCP DCR

C ⇔ c ⇔ A‡ ⇔ a‡ ⇔ G ⇔ g ⇔ E‡ ⇔ e‡ ⇔ …

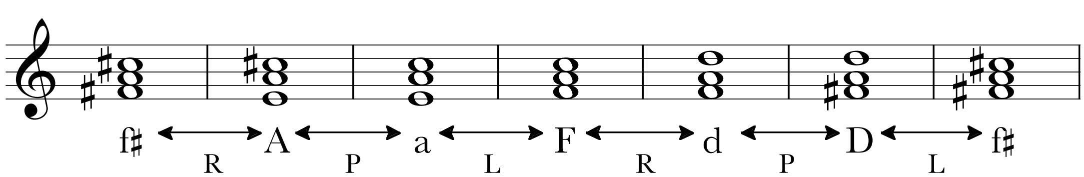
Example 6.20: DC PR-cycle

DCL DCR DCL DCR DCL DCR DCL DCR

C ⇔ e♭ ⇔ F‡ ⇔ a𝄳 ⇔ B ⇔ d ⇔ E‡ ⇔ g‡ ⇔ …

Example 6.21: DC LR-cycle

Binary transformations made up of pairings of DCP, DCL, and DCR generate cycles of DC-chords. Pairing DCP with DCR generates the eight-chord cycle shown in Example 6.19.[11] The other binary-generated cycles traverse the full set of 48 major and minor DC-chords before returning to their starting points. The root succession of the cycle generated by DCP and DCR (Example 6.20) can be separated into two alternating series of perfect fifths, {C, G, D, …} and {A‡, E‡, B‡, …}, while the root succession of the cycle generated by DCL and DCR (Example 6.21) can be separated into two alternating series of major fourths, {C, F‡, B, …} and {E♭, A𝄳, D, …}.[12]



Example 6.22: Conventional ternary cycle

[11] Coincidentally, the roots of the DC-chords in Example 6.19 are the same as the roots of the triads in Example 6.17 above. However, the DCP-DCL cycle has more in common with the conventional PL cycle than the conventional PR cycle. See Example 6.27 below.
[12] Wyschnegradsky's major fourth (int 5.5) is defined in Chapter 5. See Example 5.1.

There are three ternary transformations (PLR, LRP, and RPL) that generate cycles that return to their starting points after two iterations, each visiting six unique triads along the way.[13] Example 6.22 shows a musical realization of the RPL cycle. Each triad in the ternary-generated cycle shares a common pitch-class; in Example 6.22, the common pitch-class among all triads is A♮. Cohn observes that complete ternary-generated cycles can be found in the music of Verdi, Wagner, and Liszt.[14]

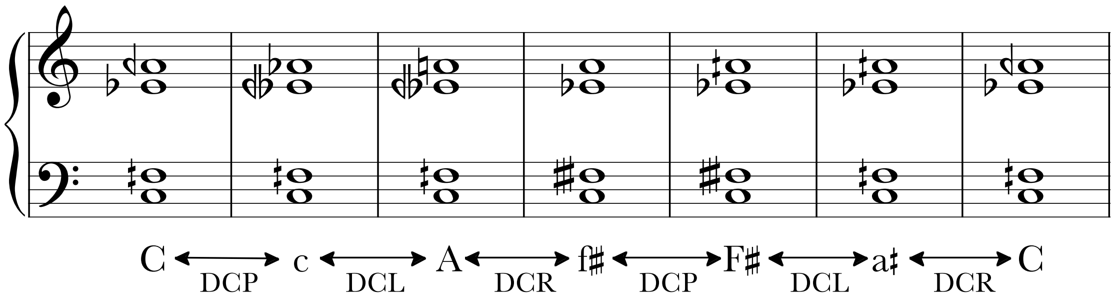
DCP DCL DCR DCP DCL DCR
C ⇔ c ⇔ A ⇔ f♯ ⇔ F♯ ⇔ a‡ ⇔ C

DCL DCR DCP DCL DCR DCP
C ⇔ e♭ ⇔ F‡ ⇔ f‡ ⇔ D‡ ⇔ c ⇔ C

DCR DCP DCL DCR DCP DCL
C ⇔ a‡ ⇔ A‡ ⇔ c‡ ⇔ E♭ ⇔ e♭ ⇔ C

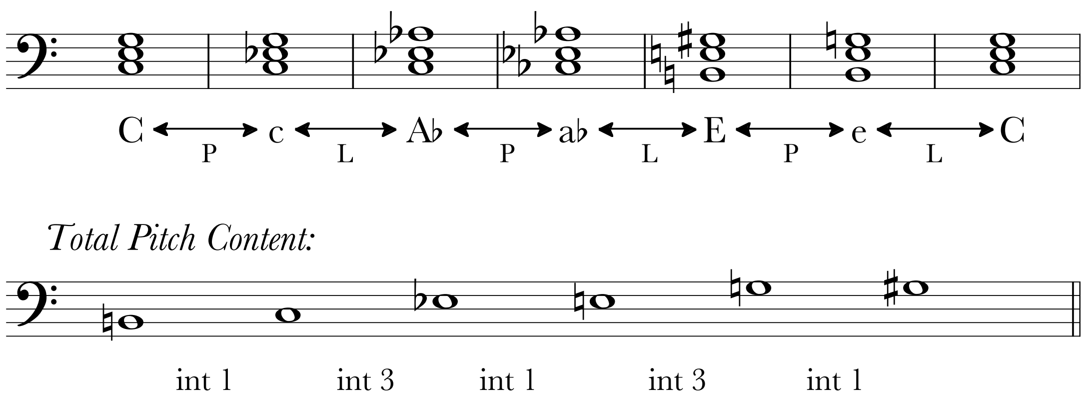
Example 6.23: Ternary cycles of DC-chords

[13] Aside from these three cycles (and their retrogrades RLP, PRL, and LPR), all other ternary combinations of P, L, and R degenerate into cycles of fewer than six triads. See Cohn, 43.
[14] Cohn, 43-45.



Example 6.24: DC ternary cycle

The DC-equivalents of the ternary-generated cycles share two characteristics with their conventional counterparts. As Example 6.23 shows, each of the three ternary transformations (DCP-DCL-DCR, DCL-DCR-DCP, and DCR-DCP-RCL) generates a cycle composed of six unique DC-chords. The six DC-chords involved in each cycle all share a common pitch-class. For example, the DC-chords in the DCP-DCL-DCR cycle that begins and ends with a C♮ DC-major chord all share the pitch-class C♮ as a common tone (Example 6.24).



Example 6.25: The hexatonic (PL) cycle

The conventional PL cycle (see Example 6.15) is a significant compositional resource for late Romantic composers such as Wagner, Franck, Liszt, Mahler, and Richard Strauss.[15] There are interesting parallels between this cycle and its DC-counterpart, generated by DCP and DCL. Example 6.25 shows a musical realization of the conventional PL cycle beginning on a C–major triad, traversing the conventional triads C–c–A♭–a♭–E–e–C. Only six of the twelve conventional pitch-classes are represented in this cycle, forming the pc-set 6-20 {B, C, E♭, E, G, G♯}. Cohn has named this set the “hexatonic” set, analogous to the name of the octatonic set; the <1, 3> alternating pattern of semitones in the hexatonic set is similar to the <1, 2> pattern of semitones in the octatonic scale.

[15] Richard Cohn, “Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions,” *Music Analysis* 15/i (1996), 9.

Because there are only four unique transpositions of the hexatonic set, there are only four unique transpositions of the complete PL cycle. Cohn has organized these transpositionally equivalent PL cycles into a hyper-hexatonic system, reproduced in Example 6.26.[16] Each of the four circles, labelled H_0, H_1, H_2, and H_3, represents one of the unique transpositions of the PL cycle that generates a specific transposition of the hexatonic set. Each hexatonic set comprises two augmented triads. For example, the hexatonic set represented by H_0 is composed of the two augmented triads {E♭, G, B} and {C, E, A♭} as shown in the center of Example 6.26. Adjacent hexatonic sets share a single augmented triad; H_0 shares {C, E, A♭} with its clockwise neighbour, H_1. Polar opposites, such as H_0 and H_2, represent complementary hexatonic sets and share no common pitches. Cohn shows that the hyper-hexatonic system has analytical relevance in some late Romantic music, citing passages from Franck, Liszt, and Wagner that progress along multiple hexatonic cycles within a short span of only a few measures.

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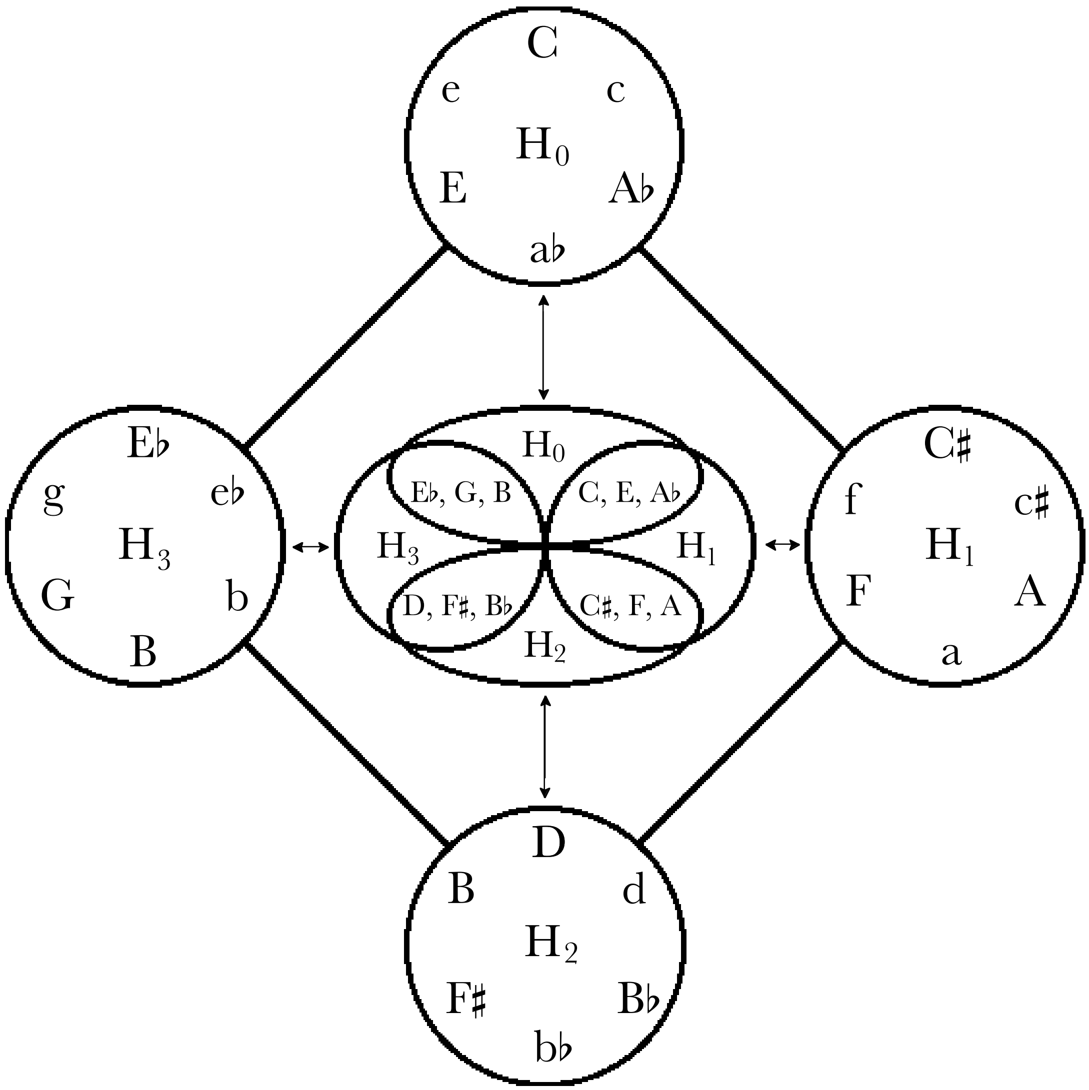
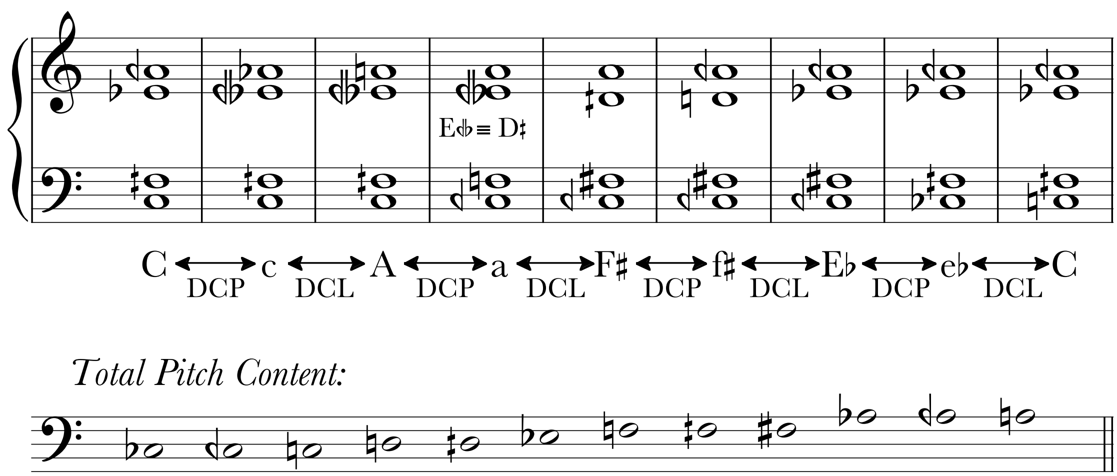
[16] Cohn, "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions," 24. My Example 6.26 reproduces Cohn's Figure 5.



Example 6.26: Cohn's hyper-hexatonic system

One can construct a system analogous to Cohn's hyper-hexatonic system using DC-chords and the transformations DCP and DCL. Example 6.27 presents a musical realization of the DCP–DCL cycle beginning on a C DC-major chord. In this cycle, only twelve of the twenty-four quarter-tone pitch classes are represented, forming the set shown on the lower staff of Example 6.27. I call this cycle the "DC-hexatonic" cycle because it was created by taking Cohn's hexatonic cycle and substituting conventional transformations and triads with their DC equivalents.[17]

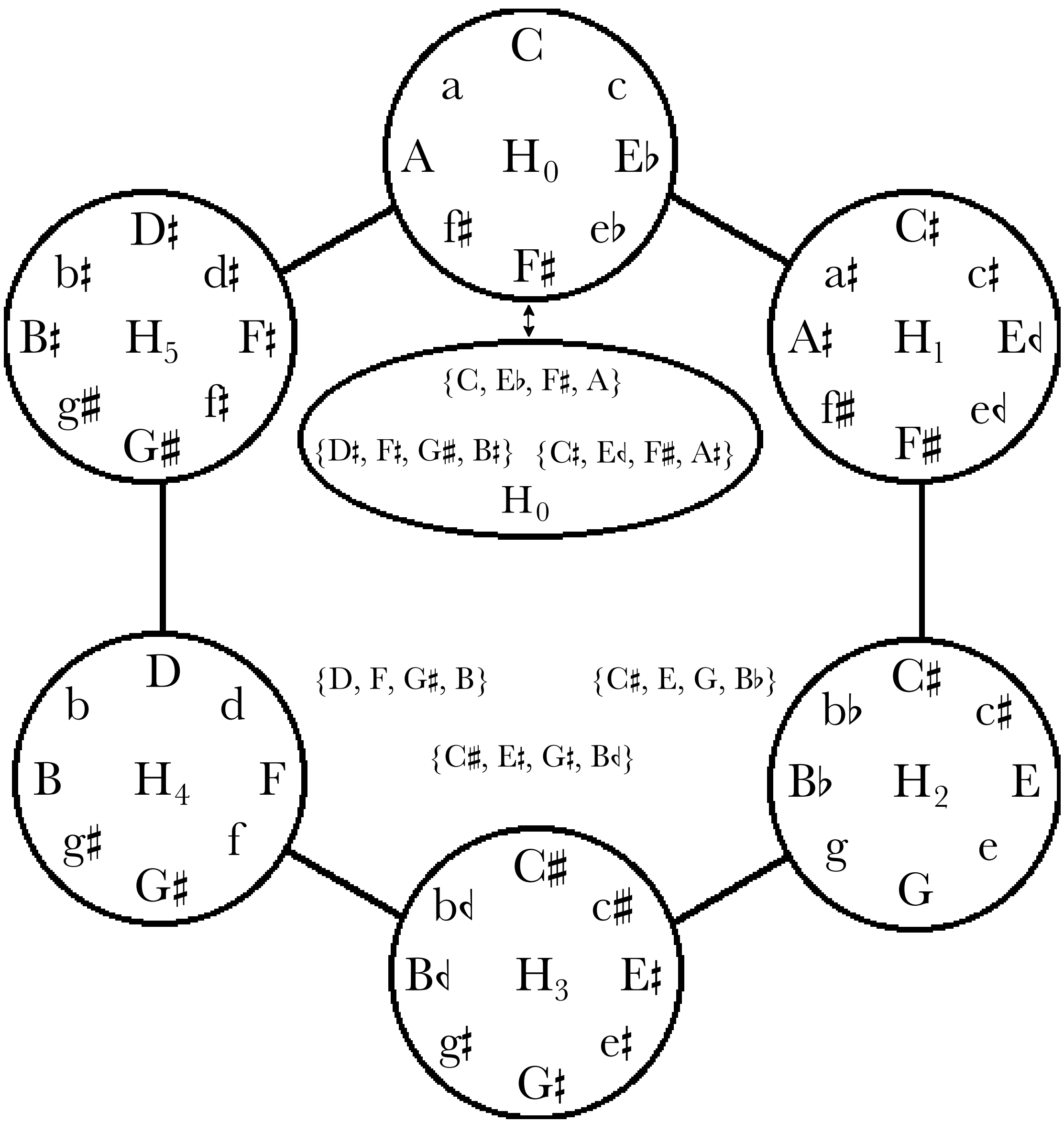


Example 6.27: DC PL-cycle

The increased pitch density caused by substituting quarter-tone tetrachords for conventional triads results in a more complex network of transpositionally equivalent cycles. As shown by Example 6.28, instead of Cohn's four cycles, there are six. In the hyper-hexatonic, each cycle's total

[17] "Dodecatonic" is a more accurate label than "DC-hexatonic" because the DCP-DCL cycle encompasses twelve pitch-classes, and not six. However, the term "dodecatonic" too closely resembles "dodecaphonic" (a term often used to describe twelve-tone music), while the "DC-hexatonic" label suggests the DCP-DCL cycle's connection with its conventional counterpart.

pitch content comprises two augmented triads; in the DC-hyper-hexatonic, each cycle's total pitch content comprises three diminished seventh chords. For example, the DC equivalent to H_0 is made up of the three diminished seventh chords {D𝄲, F𝄲, G𝄰, B𝄲}, {C, E♭, F♯, A} and {C𝄲, E𝄳, F𝄰, A𝄲}. In the hyper-hexatonic, adjacent cycles share a single augmented augmented triad; in the DC-hyper-hexatonic, adjacent cycles share two out of three tetrachords. For example, the DC-hexatonic cycles H_0 and H_1 share {C, E♭, F♯, A} and {C𝄲, E𝄳, F𝄰, A𝄲} as common tones. Non-adjacent cycles that are not polar opposites (such as H_0 and H_2) share a single diminished seventh chord. As in the hyper-hexatonic system, polar opposites (such as H_0 and H_3) represent complementary cycles that share no common pitches. Although Cohn's hyper-hexatonic system can be found in the music of some Romantic composers, I know of no composer that exploits the DC-hyper-hexatonic system. However, composers who are interested in experimenting with quarter-tone cycles featuring smooth voice-leading may find the DC-hyper-hexatonic system a promising compositional resource.



Example 6.28: DC-hyper-hexatonic system

* * *

Quarter tones have been an obvious thread connecting the diverse topics and compositions throughout this dissertation, but there has been one less obvious that is exemplified by Charles Ives's *Three Quarter-Tone Pieces.* In the 1930s, Henry Cowell identified Ives as an experimentalist, isolated from the mainstream; later scholars have attempted to situate him in the European tradition. Neither point of view captures the complete truth, however. John McGinnis identifies an "inherent conflict between tradition and invention in Ives's music."[18] To some extent, all the quarter-tone music I have studied embodies this conflict. It is easy to label any microtonal music as "experimental" because it exploits unfamiliar pitch materials. For example, Ives characterizes his own *Three Quarter-Tone Pieces* as little more than experiments (see Chapter 4), and Blackwood's *Twelve Microtonal Etudes* are literally conceived as compositional experiments, designed to demonstrate the properties of unfamiliar equal-tempered tuning systems. Although these composers' works are in some way experimental, throughout the dissertation, nearly every observation I make relies on the concepts of traditional music theory.

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[18] John McGinnis, "Has Modernist Criticism Failed Charles Ives?" *Music Theory Spectrum* 28/1 (2006), 103.

Each chapter, in fact, focuses on specific, traditional theoretical concepts. Chapter 1 considers the basic issues of music theory that all music students must learn: notation, enharmonic equivalence, intervals, chords, and scales. Chapter 2 considers counterpoint and dissonance treatment in Blackwood's *24 note*. Chapter 3 shows that motives, conventional triads, and tonal allusions are important components of Hába's *Suite für vier Pausonen im Vierteltonsystem*, op. 72. Chapter 4 shows that Ives's *Three Quarter-Tone Pieces* display stylistic features—such as symmetrical sets, interval cycles, tonal allusions, and quotation—that also appear in his more conventional works. Chapter 5 explores the characteristics of Wyschnegradsky's *diatonicized chromatic* scale (or DC-scale), and shows how he uses it in *24 Préludes dans l'echelle chromatique diatonisée à 13 sons* to create conventional tonic prolongations. And Chapter 6 further explores properties of the DC-scale and reveals parallels between relationships among DC-chords and conventional neo-Riemannian transformational theory. The material in each chapter, then, places a variety of quarter-tone concepts in the context of familiar, traditional musical concepts, casting the unknown in terms of what is conventionally known.

In general, most of the analytical issues discussed in this dissertation typify those found in analyses of conventionally-tuned music. From this perspective, my dissertation is less about "quarter-tone music" than music in general—music that happens to have been composed using the quarter-tone gamut of twenty-four pitches rather than the traditional twelve. It is easy to assume that a dissertation on microtonal music must inevitably be dense and impenetrable, concerning itself not with musical issues but with complex mathematical formulations and elaborate discussions of ratios and tuning systems. I believe that my analytical chapters provide a satisfactory answer to this misconception, for Blackwood's use of strict counterpoint, Hába's derivation of motives, form, and tonal structures, Ives's idiosyncratic stylistic features, and Wyschnegradsky's use of chord tones and prolongations all concern key musical issues frequently addressed in the context of more conventional tunings. We may conclude that what is most striking about the analysis of microtonal music is that the same sort of relationships that we care about in music of the familiar twelve-note equal temperament can also be found in unfamiliar pitch universes; we have only to listen for them.

Blackwood's search for recognizable intervals in *Twelve Microtonal Etudes* (see Chapter 2) suggests, however, that quarter tones may represent a unique tuning among microtonal schemes. Blackwood's research indicates that there is a subjective tolerance for out-of-tune intervals. Many microtonal intervals simply sound like mistuned versions of their conventional counterparts. For example, the 5:4 "pure" major third derived from the harmonic series, the 400-cent major third of 12-note equal temperament, and the flat major third from Blackwood's *19 notes* are all similar in size; although trained musicians may disagree over how well in tune these intervals are, they will generally agree that they sound "close enough" to be recognizable as major thirds. A quarter-tone interval such as int 3.5, however, lies exactly halfway between the major third and the minor third; it is distinct and cannot be recognized as either one or the other. All quarter-tone intervals likewise function as distinct intervals rather than as nuances of conventional intervals. It may be, then, that quarter tones are an ideal medium for blending the experimental with the traditional.